

Defining Complexity in the Φ Function

James F. Lynn

Abstract

This paper defines and explores the role of complexity (C) within the Φ (Phi) function, a scalar designed to measure a systems evolutionary progress. Unlike entropy or energy efficiency, complexity is a subtle but crucial factor that tracks structured, functional organization in time-evolving systems. We formally define complexity in various contexts and present a quantitative model for estimating C in a real system, such as a neural network.

1. Introduction

The Φ function describes the directional evolution of a system using three interacting quantities:

$$\Phi = \alpha \left(\frac{dC}{dt} \right) - \beta \left(\frac{dS}{dt} \right) + \gamma \left(\frac{dE}{dt} \right)$$

Where:

- $C(t)$ = Complexity
- $S(t)$ = Entropy
- $E(t)$ = Energy efficiency or coherence
- α, β, γ = Scaling weights

This paper focuses on C , the complexity term, and how it can be understood and measured.

2. What is Complexity?

In the context of Φ , complexity is:

- **Structured:** It is not noise or randomness.
- **Functional:** It supports adaptability, memory, or prediction.
- **Balanced:** It lies between rigidity (low entropy) and chaos (high entropy).

This makes complexity an emergent, functional order. High complexity implies useful internal differentiation, such as modularity or feedback systems.

3. Complexity Metric in Real Systems

Let us consider a neural network:

- N = Total neurons
- A = Active neurons (firing above threshold)
- W = Non-zero connection weights

We propose a metric:

$$C = \log_2(W) \cdot \left(\frac{A}{N}\right)$$

Example

$$N = 1000, \quad A = 250, \quad W = 5000$$

$$C = \log_2(5000) \cdot \left(\frac{250}{1000}\right) \approx 12.29 \cdot 0.25 = 3.07$$

This C value can be used dynamically in Φ over time.

4. Interpretation and Usage

- If $\frac{dC}{dt} > 0$, the system is developing more functional structure.
- If $\frac{dC}{dt} < 0$, the system is losing adaptability or order.

This makes C a key indicator in biological, artificial, and cosmological systems where structure plays a role in long-term success.

5. Conclusion

The complexity term C in the Φ function is an essential, measurable quantity representing intelligent evolution. By using domain-specific definitions of complexity, Φ becomes an actionable tool for studying adaptive systems across disciplines.